AP Calculus AB Course Syllabus

Course Design and Philosophy

It is my intent to balance conceptual understanding of the major topics of study with sufficient technical detail in order to make calculus come to life for my students.

I begin the school year by telling my students that we will study a handful of major topics: limits, derivatives, and integrals, with much time spent investigating the application of any one or any combination of those topics.

Variety of Analysis to Increase Understanding

In an effort to increase the depth of understanding for students as well as provide students with a variety of lenses through which they can analyze problems, we will emphasize the following problem-solving approaches:

Numerical methods
Graphical methods
Algebraic methods

I stress that numerical and graphical methods can be of enormous help in gaining insight and both can be highly suggestive of many things, but we must eventually bring an analytic method to bear in order to prove something.

Since many of my students concurrently take a course in Physics, whenever possible, I seek to help my students make connections between our course of study and their physics class.

Students will be expected to utilize their graphing calculator and/or the online graphing calculator found at www.desmos.com to experiment and explore problems through numerical and graphical analysis. Furthermore, students are expected to communicate any numerical and graphical analysis made possible by their graphing calculator to explain their thinking and problem solving.

APPROACHES TO TEACHING THE COURSE

1. Lecture and demonstration by the instructor coupled with discussion across the class with the intent to engage student's active participation in the learning environment.

2. Cooperative and exploratory activities for students using appropriate technology, including the TI-83+ graphing calculator, software, and web-based applications so that students may experiment with a variety of methods of analysis and reinforce concepts in conjunction with analytic methods. Facilitated by the instructor, to emphasize the active role each student holds in the learning environment.

Course Objectives (per the College Board):

- Students will be able to work with functions represented in a variety of ways: graphical, numerical, analytical, or verbal with emphasis placed upon the connections among these representations.
- Students should be able to use technology to experiment, explore, and solve problems, experiment, interpret results, and support conclusions.
- Students will be able to determine the reasonableness of solutions, including sign, size, relative accuracy, and units of measurement.
- Students will understand the derivative as both a rate of change and local linear approximation, and be able to use derivatives to solve a variety of problems.
- Students will understand the definite integral both as a limit of Riemann sums and as the net accumulation of change, and will be able to use integrals to solve a variety of problems.
- Students will understand the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus.
- Students will be able to model a written description of a physical situation with a function, a differential equation, or an integral for purposes of problem solving.
- Students will develop an appreciation of calculus as a coherent body of knowledge and an amazing human accomplishment.
- Students will be able to communicate mathematics and explain solutions to problems.

Outline of Course Content

Unit 1: **Functions** (Appendices A, B, C, D, E, F and Chapter 0)

Students complete this review of Pre Calculus topics over the summer preceding the school year that they take A. P. Calculus.

1. Functions
    a. Definition
    b. Domain and range
    c. Absolute value functions
    d. Piecewise functions

2. Properties of functions and their graphs
    a. Sums, differences, products, and quotients of functions
    b. Composition of functions
    c. Translations
    d. Stretches and compressions
    e. Symmetry
    f. Even and odd

3. Families of functions
    a. Power functions
    b. Polynomial functions
    c. Algebraic functions
    d. Trigonometric functions

4. Inverse functions
    a. Domain and range
    b. Finding inverse functions
    c. Existence of inverse function & Horizontal Line Test
    d. Graphs of inverse functions
    e. Restricting domain
    f. Inverse trigonometric functions

5. Exponential and Logarithmic functions
    a. Definitions
    b. Inverse relationship
    c. Properties of logarithms
    d. Change of base

Unit 2: Limits and Continuity (Chapter 1)

1. Limits: an intuitive approach
    a. Estimating limits from graphs
    b. Average speed versus instantaneous speed

2. Computing Limits at a point
    a. Notation
    b. 1-sided limits
    c. 2-sided limits
    d. Properties of Limits

        - A tabular exploration of the limit as *x* approaches 0 of is conducted with graphing calculator to see whether there is a limit in light of the indeterminate form .

          

          Students then look at the graph of to see whether or not there is agreement with the tabular results. This is aimed at communicating to students how the idea of "closeness" is at the heart of a limit rather than what occurs when . Finally, we examine the limit through the lens of the Squeeze Theorem.
        - Students will, especially early on, will use a graphing calculator to estimate limits and to understand a variety of important limit facts for a variety of functions.

3. Computing Limits involving infinity
    a. Asymptotic behavior
    b. End behavior models

        - A tabular investigation of the limit as *x* approaches of is conducted in exploration of whether there is a limit in light of the indeterminate form . We then look at the graph of to see whether or not there is agreement with the tabular results. A graphing calculator is used to examine the graph as *x* approaches . Finally, we examine the limit analytically by dividing the terms of the numerator and denominator by and using basic properties of limits.

- Students will use graphing calculators to explore the asymptotic behavior of certain functions and how that relates to specific limits for that function.

4. Continuity
    a. At a point and over an interval
    b. Continuous functions
    c. Discontinuous functions
    d. Types of discontinuities

- A tabular investigation of the limit as *x* approaches 1 of is conducted in exploration of whether there is a limit in light of the indeterminate form . We then look at the graph of to see whether or not there is agreement with the tabular results. Students then examine the limit analytically by removing the common factor and explain why the function is discontinuous at using the definition of continuity.

- Students will also use graphing calculators to explore the continuity of functions.

- Students are asked to sketch graphs for functions that satisfy given conditions relating to continuity and explain to their classmates how they determined the appropriateness of their example using the definition of continuity. For example, students must sketch …

1) a function *f* that is continuous everywhere except x = 3 at which it is continuous from the right.

2) a function *f* that has a two-sided limit at x = 3, but is not continuous at x = 3.

Students then share and compare their examples of functions satisfying the given conditions.

5. Limits and Continuity of Trigonometric, Exponential, and Inverse Functions

Unit 3: The Derivative (Chapter 2)

1. Slopes and Rates of change
    a. Average rate of change as the slope of the secant line
    b. Instantaneous rate of change as the limiting value for the slope of the secant line

2. The Derivative
    a. Definition of the derivative as a limit of a difference quotient
    b. Derivative at a point
    c. Estimating the derivative from tabular data & appropriate difference quotient
    d. Graphing from , graphing from
    e. One-sided derivatives

    - We examine varying position versus time curves and sketch velocity versus time curves to better understand the relationship between .

    - Students examine the graph of and estimate the slope of the tangent line for various values over and conjecture what trigonometric function the graph of happens to be.

    - Students are asked to determine the derivative for , , , , , and early in this unit before we cover basic differentiation rules. Then they are asked to formulate a conjecture regarding a rule for differentiating functions of this type and about the derivative of a constant. Finally, students are asked to explain, both verbally and in writing, to one another what lead them to their conjecture.

3. Differentiability and continuity

a. failing to exist at corner points and points of vertical tangency
b. Differentiability implies continuity
c. Continuity does not imply differentiability

4. Techniques of Differentiation
    a. Constant, power, sum, difference rules
    b. Product and quotient rules
    c. Higher order derivatives
    d. Functions as solution of differential equations
    e. Velocity and acceleration functions as first and second derivatives of a position function

5. Derivatives of Trigonometric Functions

6. The Chain Rule

Unit 4: Derivatives of Exponential, Logarithmic, and Inverse Trigonometric Functions (Chapter 3)

1. Implicit Differentiation using differentials and

2. Derivatives of Logarithmic Functions

3. Derivatives of Exponential and Inverse Trigonometric Functions

4. Related Rates

5. Local Linear Approximation & differentials

6. L'Hopital's Rule; Indeterminate Forms

Unit 5: The Derivative in Graphing and Applications (Chapter 4)

1. Analysis of Functions I: Increasing, decreasing, and concavity

2. Analysis of Functions II: Relative extrema; graphing polynomials; First and Second Derivative Tests

3. Analysis of Functions III: Rational functions; cusps and vertical tangents

4. Rectilinear Motion

5. Absolute Maxima and Minima

6. Applied Maximum and Minimum Problems

7. Rolle's Theorem; Mean-Value Theorem

    - Students explore the graphs of various functions such as , , and using a graphing calculator. Students then discuss what each graph communicates about relative extrema, inflection points, asymptotes, and end behavior. Students follow this with analytic explanation for those characteristics noticed from the graph using formal language and symbolism.

Unit 6: Integration (Chapter 5)

1. An overview of the Area Problem
    a. Rectangle method
    b. Antiderivative method

2. The Indefinite Integral
    a. Properties of the indefinite integral

3. Integration by Substitution

4. Sigma Notation, Area as a Limit
    a. Endpoint approximation; left sums and right sums
    b. Midpoint sums
    c. Trapezoidal sums

5. The Definite Integral
    a. As a limit of Riemann Sums
    b. Properties of the definite integral

6. The Fundamental Theorem of Calculus
    a. Part 1 and part 2
    b. Mean Value Theorem for Integrals

7. Differentiation and integration are inverse processes

8. Rectilinear Motion Revisited; Average value of function

9. Evaluating Definite Integrals by Substitution

10. Logarithmic Functions from the Integral Point of View

Unit 7: Applications of the Definite Integral in Geometry, Science and Engineering (Chapter 6)

1. Area Between Two Curves

2. Volumes by Slicing; Disks and Washers

3. Volumes by Cylindrical Shells

4. Length of a Plane Curve

5. Area of a Surface of Revolution

Unit 8: Principles of Integral Evaluation (Chapter 7)

1. An overview of integration methods

2. Integration by Parts

3. Trigonometric Integrals

4. Trigonometric Substitution

Major Text Used

Anton, H., Bivens, I., Davis, S., *Calculus: Early Transcendentals Single Variable*, 9th ed. John Wiley & Sons, Inc.